Dynamic Eshelby Micromechanics

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Abstract

“Eshelby micromechanics” can be called the micromechanics based on the two celebrated Eshelby papers: The “force on an elastic singularity”, or “Eshelby force” (1951), (associated with Noether’s theorem and conserved integrals) and the ellipsoidal inclusion with transformation strain (1957)which has the constant interior constrained strain (Eshelby) property. The constant Eshelby Tensor allows for the solution of inhomogeneities by the Eshelby equivalent inclusion method. The Eshelby properties are expanded here to self-similarly expanding ellipsoids. By dimensional analysis it is shown that for self-similarly expanding inclusions the system of governing equations of elastodynamics becomes elliptic and that the particle velocity is zero in the interior domain of the expanding inclusion. From this follows that the Eshelby property also holds in the self-similar dynamic case (subsonic). The Dynamic Eshelby Tensor for *self-similarly* expanding ellipsoidal inclusions is obtained and is constant (depending on the wave and axes expansion speeds), and this allows for the solution of expanding inhomogeneities. The static Eshelby Tensor is obtained from the dynamic one by a limiting procedure.

The elastodynamic evolution of moving defects (dislocations, expanding inclusion and inhomogeneity boundaries) is governed by the dynamic conservation laws (*J, L, M* integrals) from Noether’s theorem yielding the “kinetic relations” due to inertia. For a solid containing a periodic distribution of defects, in the unit cell the defects evolve by Eshelby mechanics (J, L, M integrals) and this is carried to the macroscopic scale as macroscopic damage by asymptotic homogenization. The self-similar dynamic solutions for expanding inclusions/inhomogeneities grasp the early response of the system, and have applications to dynamically stress induced martensitic transformations and the modeling of deep earthquakes.